Aktivität 3.4. Die Natur der Geometrie und die Geometrie der Natur

Cultural Background

Seit Jahrtausenden bemüht sich die menschliche Zivilisation, die perfekte Geometrie in der Natur zu verstehen. Wir alle haben schon als Kinder die wunderschönen Strukturen der Schneekristalle bewundert. Es ist wie ein echter Zauber. Perfekte Fraktale, exakte Symmetrie – wie schafft die Natur so eine Vollkommenheit in den Formen zu erreichen? Versuchen Sie nächstes Mal, wenn Sie in der Natur sind, geometrische Formen zu finden. Sie werden überrascht sein, was für eine Vielfalt sich vor Ihren Augen entfaltet.

In *Waldsee*

In *Waldsee*'s STEM high school credit program, students explore the correlation between surface area and volume, first by creating cylinders with sheets of paper and determining what relationship and/or correlation exists for surface area and volume. Then students apply their hypotheses to local trees, measuring sections of their trunk (to maintain cylinders) and then explaining the dimensions required of the various different trees to maintain the same volume. Once their investigation of cylinders is complete, they repeat the same structure with triangular prisms, initially investigating Toblerone bars and their relative dimensions. Then on nature walks they find examples of natural triangular prisms, comparing their associated measurements, determining the proportion of size between two different plants in the same way as with the Toblerone chocolate.

In the Classroom

In this activity students combine their knowledge of German and geometry to explore the correlation between surface area and volume in geometrical shapes found in nature. Initial investigations can be conducted in the classroom, and can be expanded to natural geometrical shapes found outdoors.

Objectives

- **Communication**
    - Students will use German language constructions and vocabulary to compare and contrast geometrical forms found in nature.
    - Students will employ German language constructions and vocabulary to use geometric equations.
    - Students will master subject-specific terminology and vocabulary and read, interpret, and solve a variety of geometry questions in German related to their environment.
    - Determining methods for measuring dimensions of various local ecology.
    - Students will formulate questions and justify answers in German.
    - Students will explain a logical sequence of events and/or steps in order to conduct measurements and explain thinking.
    - Students will use if/then statements for explanations involving height/width changes and the associated changes (or lack thereof) in surface area and/or volume of the related figures.

    - Students will create and present a summary of results, in 5+ sentences, both verbally and in writing.
    - Students further their understanding of dependent clauses and syntax, and independent clauses and their associated syntax, especially in their use of sequential/methodical explanations

- **Connections**
    - Students will reinforce and integrate their knowledge of math and environmental studies through use of German.
    - Students will be able to make connections to their natural surroundings through the German language by generating, justifying and validating hypotheses.
    - Students will deepen their connections to their physical environment and better identify local plants and wildlife, with a new perspective -- and further solidify their understanding of geometric shapes in nature.

- **Cultures**
    - Students will learn about variance in standard measuring units in the United States versus the measurement system used in Germany/Austria/Switzerland.

- **Comparisons**
    - Students will be able to use their German to compare and contrast different geometric forms and discuss various hypotheses about the results.

Language Functions in Focus

- Understanding general vocabulary and technical language
- Indicating agreement and disagreement
- Explaining processes and procedures
- Inferring and interpreting data (verbally and in writing) in the target language
- Expressing and justifying an opinion
- Comparing and contrasting
- Evaluating
- Reporting

Materials

- Paper, straight edge, measuring tape/ruler
- String, writing utensil
- Poster paper, markers, colored pencils, etc.
- Reference sheet with surface area and volume general equations for triangles, circles, quadrilaterals, and their associated prisms.

Preparation

Students should be familiar with the concepts of surface area and volume. They should be comfortable simplifying and solving equations, and with the dimensions of circles, triangles, and quadrilaterals. It also helps greatly if they understand the term base in the geometric sense, and what it means for a figure to have two bases (parallel congruent sides). They should understand the terms radius, diameter, circumference, height, width, base, and base area.

Generating Interest

Students begin with two sheets of paper (for example, A4 paper) of the same size, and are asked to create two different cylinders, without overlapping the paper at any point, nor cutting away any portion of the paper. They should find one that has a base circumference of 210mm and a height of 270mm, while the other has a circumference of 270mm and a height of 210mm. This launches into an investigation of surface area and volume, which are calculated by the students for both figures, and then generalizations about the connection between surface area and volume are generated and discussed.

Presentation and Practice

In this activity students use their German and knowledge of geometry to explore the correlation between surface area and volume, initially with cylinders, and then move on to other shapes (triangular prisms, cubes, and rectangular prisms), including those found in natural environments.

We begin our investigation with cylinders, because of the great number of trees found throughout the Waldsee village site--they are readily available, and students can measure and calculate surface area and volume from multiple trees each, without repetition of area or specific trees. This allows students to have a broader data set to analyses, and to further generalize their results, increasing the likelihood for consistency and accuracy with their results analysis.

The teacher begins by asking students to create creating cylinders with sheets of paper. "*Hier haben wir einen Zylinder. Das ist eine dreidimensionale Figur. Wenn wir an zweidimensionale Figuren denken, was für Figuren sehen wir hier in unserem Zylinder?" (Hoffentlich merken die Schüler, dass einen Zylinder aus zwei Kreisen und einem Rechteck erstellt werden kann.)* Next, students will work to determine what relationship and/or correlation exists for surface area and volume for their cylinders. "*Wie messen wir den Umfang dieses Rechtecks? Wie messen wir den Umfang eines Kreises? Der Umfang ist ein zweidimensionales Konzept; in drei Dimensionen, möchten wir statt den Umfang, die Flächeninhalt berechnen. Um sie zu finden, brauchen wir die Flächen von unseren Kreisen und Rechteck zusammen zu addieren.*" Then the teacher gives the students time to work on finding the individual areas of their constructed shapes, and then to calculate the surface area of their cylinders. "*Als nächstes, möchten wir wissen, wie groß der Innenraum des Zylinders ist. Das heißt das Volumen. Um das zu berechnen, brauchen wir die Fläche unserer Grundseite, in diesem Fall einen Kreis. Danach brauchen wir die Höhe der Figur. Die Höhe multiplizieren wir mit der Fläche, und unser Resultat ist dann das Volumen.*" Then the teacher then gives the students time to work on finding the individual volumes of their constructed cylinders.

Next, the teacher takes the class outside and asks students will try to apply their generalization hypotheses to local trees, measuring sections of their trunk (to maintain cylinders) and then explaining the dimensions required of the various different trees to maintain the same volume. Students photograph the tree trunks, and annotate the areas to which they limited their measurements on the photos, in addition to a written comparison and explanation of their work.

Once their investigation of cylinders is complete, they repeat the same structure with triangular prisms, initially investigating the relative dimensions of Toblerone chocolate bars. In this case, instead of looking for dimensions to maintain volume, they look at proportions. The teacher asks students to determine the surface area and volume for a regular-sized Toblerone, and then if the face dimensions are increased by a factor of x, to determine by which factor the volume and the surface area increases.

Following these activities, the class takes a nature walk, and students are asked to find examples of triangular prisms in their own natural environment. Once they find some examples, the teacher asks them to compare their associated measurements, determining the proportion of size between two different plants in the same way as with the Toblerone. For this particular shape, students are likeliest to find examples in rocks, or perhaps in tents. If students are unable to find triangular prisms on their walk, another option is to use food, such as a slice of watermelon, to model triangular prisms.

Finally, students repeat the same experimental idea with either a cube or a rectangular prism, whichever they prefer, and measure and determine both, two items that have either the same volume/surface area given different dimensions, and the proportionate increase in surface area/volume by two similar but incongruent shapes.

If students are unable to conduct their activities outside, they could find examples of the same shapes either by research, if given access to the internet, or in their own classroom/building/home environment. The comparisons, related diagrams, equations, and explanations, would all remain the same. A third and/or fourth demonstration, with a cube (for example, a set of dice) and a rectangular prism (shipping boxes, textbooks, cereal boxes, etc.) could be used for the initial investigation.

Wortschatz

der Kreis	*symmetrisch*	*suchen*
die Ecke	*parallel*	*die Gleichung*
die Struktur	*die Messung*	*die Variable*
der Winkel	*die Höhe*	*die Unbekannte*
das Rechteck	*die Breite*	*die Formel*
das Quadrat	*messen*	*der Flächeninhalt*
der Zylinder	*bestimmen*	*das Volumen*
das Dreieck	*rechnen*	*zuerst*
die Fläche	*erklären*	*als nächstes*
die Seitenfläche	*begründen*	*später*
der Flächeninhalt	*erweitern*	*danach*
die Grundseite	*feststellen*	*als letztes*
die Seite	*forschen*	*zuletzt*

Expansion

<u>*Zur Diskussion -- Frage und Antwort:*</u>

1. *Wie viele Ecken weist die Struktur eines jeden Schneekristalls auf?*
   a. *zwölf*
   b. *sechs*
   c. *es gibt keine feste Anzahl von Ecken*
   d. *fünf*

(Richtige Antwort: sechs. Obwohl die Formenvielfalt der Schneekristalle gewaltig ist, haben alle eine Gemeinsamkeit: Sie sind immer sechseckig. Wissenschaftler führen das auf die molekulare Struktur des Wassers zurück, die bei der Kristallbildung nur Winkel von exakt 60° bzw. 120° zulässt.)

2. *Die meisten Lebewesen sind bilateralsymmetrisch. Was bedeutet das?*
    a. *Ihr Körper hat eine vordere und eine hintere Hälfte, die zueinander spiegelsymmetrisch sind*
    b. *Sie haben eine linke und eine rechte Hälfte, die zueinander spiegelsymmetrisch sind*
    c. *Ihr Körper hat eine obere und eine untere Hälfte, die zueinander spiegelsymmetrisch sind*

(Richtige Antwort: Ihr Körper hat eine linke und eine rechte Hälfte, die zueinander spiegelsymmetrisch sind. In der Biologie wird von Bilateralität gesprochen, wenn die linke und rechte Hälfte eines Organismus zueinander spiegelsymmetrisch sind. Rund 95 Prozent der vielzelligen Tiere folgen dieser Grundform.)

3. *Gibt es Rechtecke oder Quadrate in der Natur?*
    a. *Rechtecke kommen in der Natur vor, Quadrate nicht*
    b. *Jein. Alle in der Natur vorkommenden Rechtecke oder Quadrate verdanken ihre Form dem Zufall*
    c. *Ja, Rechtecke und sogar Quadrate kommen in der Natur vor*
    d. *Nein, echte Rechtecke oder Quadrate sind in der Natur nicht vorgesehen*

(Richtige Antwort: Ja, Rechtecke und sogar Quadrate kommen in der Natur vor. Während natürlich entstandene Rechtecke in der Tier- und Pflanzenwelt dem Zufall zu verdanken sein dürften, sind sie in der Welt der Mineralien keine Ausnahmeerscheinung.)